

# The innocuousness of adiabatic instabilities in coupled scalar field-dark matter models

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**Abstract.** Non-minimally coupled scalar field models suffer of unstable growing modes at the linear perturbation level. The nature of these instabilities depends on the dynamical state of the scalar field. In particular in systems which admit adiabatic solutions, large scale instabilities are suppressed by the slow-roll dynamics of the field. Here we review these results and present a preliminary likelihood data analysis suggesting that along adiabatic solutions coupled models with coupling of order of gravitational strength can provide viable cosmological scenarios satisfying constraints from SN Ia, CMB and large scale structure data.

**Keywords:** Cosmology-Structure Formation-Linear Perturbations

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## INTRODUCTION

The possibility of a direct coupling between a quintessence-like scalar field and the various matter components has been extensively studied in a vast literature (see e.g. [1]). Non-minimally coupled scalars appear in various theoretical scenarios which attempt to describe fundamental interactions at energies beyond that of the Standard Model of particle physics (see e.g. [2]). Their application to cosmology and the unsolved problem of dark energy in the universe has suggested a number of interesting features, most importantly the solution of the so called coincidence problem. The presence of the scalar interaction is not incompatible with existing constraints on the violation of the Equivalence Principle. Ingenuous mechanisms (which may differ from one model to another) guarantee that standard General Relativity is recovered on Solar System scales (e.g. [3, 4]), and leave distinctive signatures on the structure formation process, eventually contributing to some of the still not understood phenomena in the context of Cold Dark Matter paradigm [5]. Consequently the distribution of structures, at least on those scales where observations have provided accurate measurements, is a key test that such models have to pass. Nevertheless a number of works have indicated that coupled scalar field models may suffer of large scale instabilities at the linear perturbation level. This was initially pointed out in some specific realizations [6] and recently discussed in more general setups [7, 8]. The claim has been particularly emphasized on scenarios characterized by the existence of the so called “adiabatic” regime, such as the Chameleon model [7]. In the light of these results coupled models seem to be unrealistic cosmological scenarios. However as we have shown in [9] the instabilities are not generic, rather they are strongly dependent on the dynamical state of the scalar field. More importantly along “adiabatic” solutions and for natural values of the coupling constant such instabilities are innocuous. Here we will briefly summarize the main results of [9] to which we refer the reader for a more detailed discussion. We will also present the results of a preliminary likelihood data analysis to test the viability of these models against current cosmological observations.

## PERTURBATIONS IN COUPLED SCALAR FIELD-DARK MATTER MODELS

Let us consider a scalar field  $\phi$ , with potential  $V(\phi)$ , coupled to matter particles through a Yukawa coupling of the form  $f(\phi/M_{Pl})\psi\bar{\psi}$ , where  $f$  is the coupling function and  $\psi$  is the Dirac spinor associated with the matter particle ( $M_{Pl} = 1/\sqrt{8\pi G}$  with  $G$  the Newton constant). For simplicity let us consider the case in which the scalar field is coupled to dark matter only, thus Equivalence Principle constraints are immediately satisfied. This is not a restrictive assumption since our results can be extended also to models with couplings to all matter components provided the existence of an “adiabatic” regime. Because of the coupling the energy-momentum tensor of each component of the system is not conserved. It is only the total energy-momentum tensor that satisfies the conservation equation:

$T_{V;\mu}^{\mu(T)} \equiv T_{V;\mu}^{\mu(\phi)} + T_{V;\mu}^{\mu(DM)} = 0$ . Now let us consider a coupling function of dilatonic type,  $f(\phi) = \exp(\beta\phi/M_{Pl})$ , with  $\beta$  the dimensionless coupling constant, from the above conservation condition in a flat Friedmann-Lemaitre-Robertson-Walker background ( $ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2$ ) we obtain the evolution equations:

$$\dot{\rho}_{DM} + 3H\rho_{DM} = \beta\dot{\phi}\rho_{DM}, \quad (1)$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = -\beta\rho_{DM}, \quad (2)$$

with the Hubble rate given by  $H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}[\rho_{DM} + \dot{\phi}^2/2 + V(\phi)]$ . The solution to Eq. (1) reads as

$$\rho_{DM} = \frac{\rho_{DM}^{(0)}}{a^3} e^{\beta(\phi - \phi_0)}, \quad (3)$$

where  $\phi_0$  is the present scalar field value. From Eq. (1) and Eq. (2) we may notice that for positive values of the coupling constant  $\beta$ , the interaction transfers energy from the  $\phi$ -field to the dark matter particles, with the scalar field evolving in an effective potential

$$V_{\text{eff}}(\phi) = V(\phi) + \frac{\rho_{DM}^{(0)}}{a^3} e^{\beta(\phi - \phi_0)}, \quad (4)$$

characterized by a minimum

$$V_{,\phi_{\text{min}}} = -\beta \frac{\rho_{DM}^{(0)}}{a^3} e^{\beta(\phi_{\text{min}} - \phi_0)}. \quad (5)$$

Given the above background equations, the evolution of linear density fluctuations can be studied by perturbing the Einstein equations and the conservation of the total energy momentum tensor about a linearly perturbed FLRW background. However in order to gain some intuitive insight on the behavior of the perturbations on the large scales and perform a simple stability analysis, it is convenient to consider the interacting scalar field-dark matter system as an effective single fluid with energy density  $\rho_T = \dot{\phi}^2/2 + V(\phi) + \rho_{DM}$ , pressure  $p_T = \dot{\phi}^2/2 - V(\phi)$ , and whose perturbations are uniquely characterized by an adiabatic sound speed,  $c_{aT} = \sqrt{\dot{p}_T/\dot{\rho}_T}$ , and the rest frame sound speed,  $c_{sT} = \sqrt{\delta p_T/\delta \rho_T}$ .

In synchronous gauge the perturbation equations reads as

$$\begin{aligned} \dot{\delta}_T &= -3H(c_{sT}^2 - w_T)\delta_T + \\ &- (1 + w_T) \left\{ \left[ \frac{k^2}{a^2 H^2} + 9(c_{sT}^2 - c_{aT}^2) \right] \frac{aH^2}{k^2} \theta_T + \frac{\dot{h}}{2} \right\}, \end{aligned} \quad (6)$$

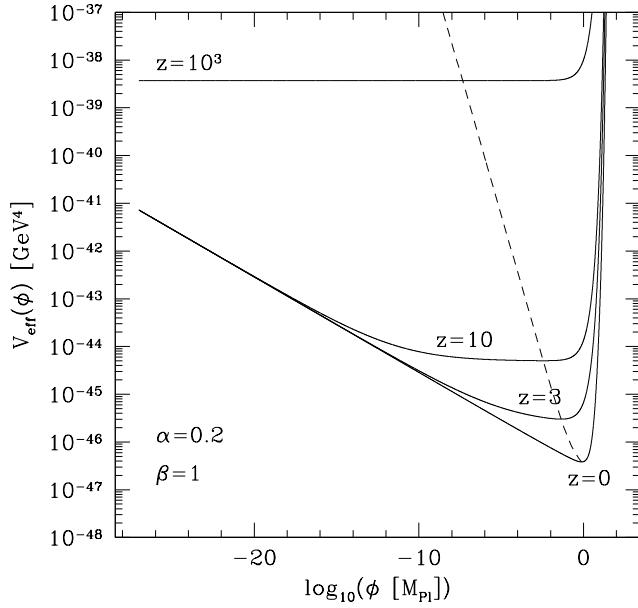
$$\dot{\theta}_T = -H(1 - 3c_{sT}^2)\theta_T + \frac{c_{sT}^2 k^2}{a(1 + w_T)} \delta_T, \quad (7)$$

where  $\delta_T = \delta\rho_T/\rho_T$  and  $\theta_T$  is the shear velocity perturbation of the fluid. For a barotropic component with a constant equation of state (e.g. matter, radiation)  $c_s^2 = c_a^2 = w$ . This is not the case for a generic fluid (e.g. scalar field), for this reason we may expect the effective unified fluid to be non-barotropic, (i.e.  $c_{sT}^2 \neq c_{aT}^2 \neq w_T$ ). In terms of the scalar field and dark matter perturbation variables we have

$$c_{aT}^2 = \frac{3H\dot{\phi}^2 + \dot{\phi}[2V_{,\phi} + \beta\rho_{DM}]}{3H\dot{\phi}^2 + 3H\rho_{DM}}, \quad (8)$$

$$c_{sT}^2 = \frac{\dot{\phi}\delta\dot{\phi} - V_{,\phi}\delta\phi}{\dot{\phi}\delta\dot{\phi} + V_{,\phi}\delta\phi + \rho_{DM}\delta\rho_{DM}}. \quad (9)$$

These relations provide us with a simple way of determining the stability of the perturbations in the coupled system, for example in a given background regime instabilities may develop if these sound speeds acquire sufficiently negative values.



**FIGURE 1.** Scalar field effective potential at  $z = 0, 3, 10$  and  $10^3$ , the dashed line shows the position of the minimum as function of the redshift.

## SCALAR FIELD DYNAMICS AND INSTABILITY ANALYSIS

An attractor solution of the background homogeneous system consists of the field seating at the minimum of the potential, and drifting in time according to Eq. (5). This is usually referred as “adiabatic” regime. It has been shown in [10] that along this solution the field slow-rolls, thus it has a negligible kinetic energy. In particular for a power law potential,  $V(\phi) \propto \phi^{-\alpha}$ , the evolution of the scalar field given by the condition Eq. (5) reads as:

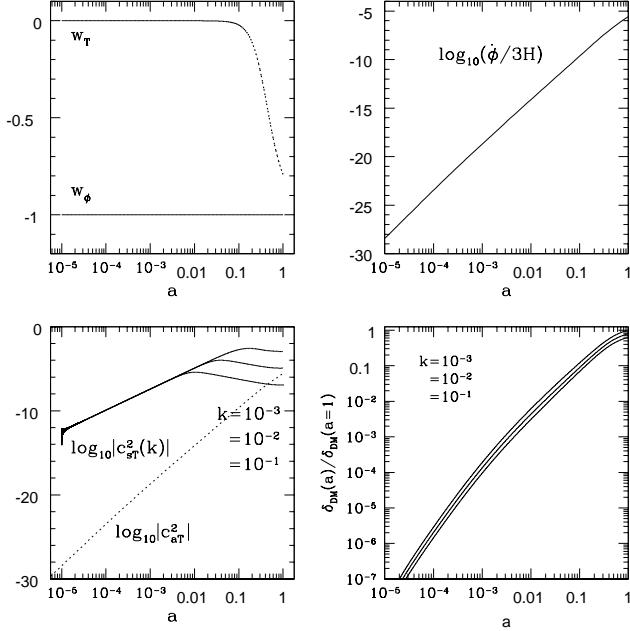
$$\left(\frac{\phi_0}{\phi_{\min}}\right)^{\alpha+1} = \frac{1}{a^3} e^{\beta(\phi_{\min} - \phi_0)}, \quad (10)$$

which depends on both the slope  $\alpha$  and the coupling  $\beta$ . Equation (10) is a non-linear algebraic equation which can be solved numerically through standard bisection methods. The presence of the minimum distinguishes two different sets of initial conditions:  $\phi_{ini} < \phi_{\min}^{ini}$  (small field) or  $\phi_{ini} > \phi_{\min}^{ini}$  (large field). For small field values,  $\phi$  evolves over the inverse power-law part of the effective potential, where it minimizes the potential by slow-rolling as shown in [10]. In contrast for initially large field values,  $\phi$  rolls towards the minimum along the steep exponential part of the effective potential. Thus it rapidly acquires kinetic energy which subsequently dissipates through large high-frequency damped oscillations around the minimum. The growth of the linear perturbations in the coupled scalar field-dark matter system is significantly different in these two regimes.

### Adiabatic Regime: slow-roll suppression of instabilities

Let us evaluate the adiabatic and rest frame sound speeds along the adiabatic solution respectively. Substituting Eq. (5) in Eq. (8) and neglecting the term proportional to the kinetic energy of the scalar field (due to the slow-roll condition) we have

$$c_{aT}^2 = -\beta \frac{\dot{\phi}}{3H}, \quad (11)$$



**FIGURE 2.** Upper left panel: evolution of the scalar field equation of state  $w_\phi$  and effective unified fluid equation of state  $w_T$ ; Right upper panel: evolution of the scalar field velocity with respect to the Hubble rate; Lower left panel: redshift evolution of the adiabatic sound speed  $c_{aT}^2$  and propagation of pressure perturbations  $c_{sT}^2$ ; Right lower panel: Linear growth factor of the dark matter density contrast at  $k = 10^{-3}, 10^{-2}$  and  $0.1 \text{ Mpc}^{-1}$ .

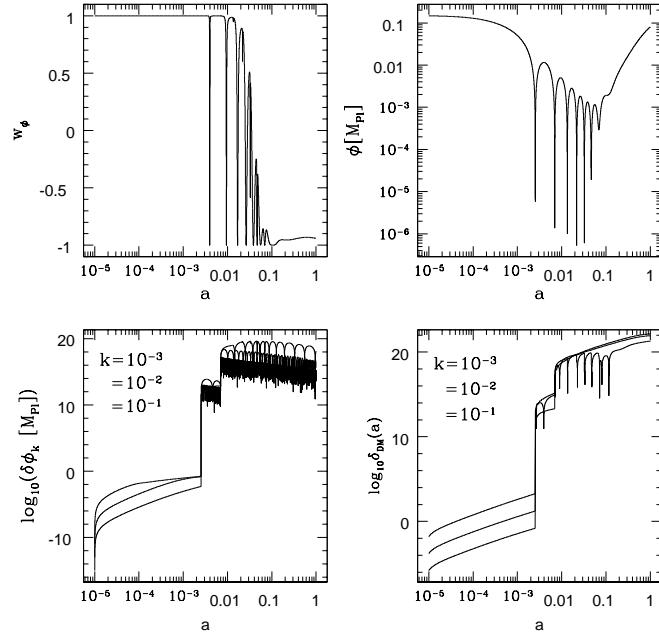
since  $\dot{\phi} > 0$  it then follows that  $c_{aT}^2 < 0$ , implying that adiabatic instabilities may indeed develop. However we should remark that during the adiabatic regime the field is slow-rolling (i.e.  $3H\dot{\phi} \approx 0$ ), hence the term  $\dot{\phi}/3H$  can be negligibly small compared to  $\beta$ , such that  $c_{aT}^2 \approx 0^-$ , thus leading to a stable growth of the large scale perturbations. In fact let us suppose that  $c_{aT}^2 = -10^{-5}$ , the instability will affect modes  $k \geq 10^5$ , but these correspond to very small scales which are already in the non-linear regime and for which the linear perturbation theory does not apply any longer. In contrast large scale instabilities will occur if the coupling assumes extremely large values,  $\beta \gg 3H/\dot{\phi}$ . This is consistent with the conclusions of [7], where the authors have shown that during the adiabatic regime perturbations suffer of instabilities provided that  $\beta \gg 1$ . However such situation would be extremely unnatural introducing a large hierarchy problem in the gravitational sector since it would implying having a scalar fifth-force which is  $(1 + 2\beta^2)$  greater than gravitational strength. Guided by naturalness considerations one might expect that the dimensionless coupling constant is of order of unity. Let us now evaluate the sound speed in the total effective fluid rest frame, Eq. (9) we have

$$c_{sT}^2 = -\frac{1}{1 - \frac{1}{\beta} \frac{\delta_{DM}}{\delta\phi}}, \quad (12)$$

assuming that the scalar field is nearly homogeneous,  $\delta\phi \ll \delta_{DM}$  (in Planck units), we have  $c_{sT}^2 \approx \beta \delta\phi / \delta_{DM}$ , and for  $\beta \approx \mathcal{O}(1)$  this implies  $c_{sT}^2 \approx 0$ . In other words if the scalar field fluctuations are small with respect to the dark matter density contrast, then the coupled system behaves has a single adiabatic inhomogeneous fluid ( $c_{sT}^2 \approx c_{aT}^2 \approx 0$ ). These results are supported by the numerical study of the perturbation equations for the individual components of the system as summarized in Fig. 2.

### Non-Adiabatic Regime: large field oscillations and onset of instabilities

For initially large field values,  $\phi$  rolls along the steep exponential part of the effective potential. Its evolution is therefore dominated by the kinetic energy and the field behaves as a stiff fluid ( $w_\phi = 1$ , as can be noticed in the left

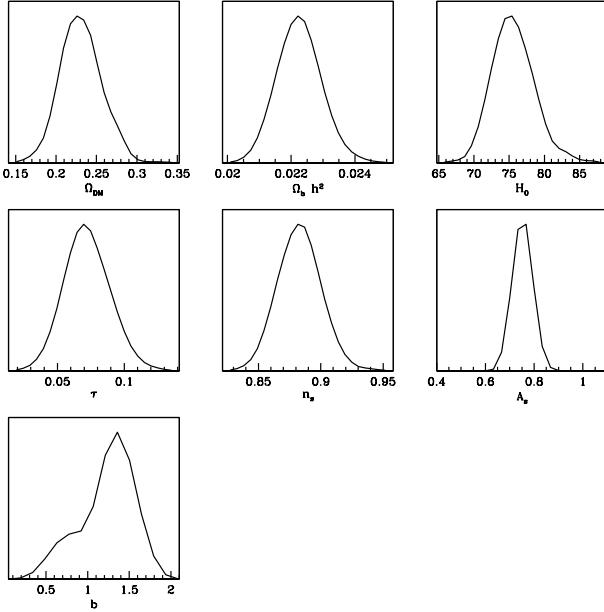


**FIGURE 3.** Upper left panel: evolution of the scalar field equation of state  $w_\phi$ ; Right upper panel: evolution of the scalar field; Lower left panel: evolution of the field fluctuations  $\delta\phi_k$  at  $k = 10^{-3}, 10^{-2}$  and  $0.1 \text{ Mpc}^{-1}$  respectively; Right lower panel: evolution of dark matter density for  $k$ -values as in the case of  $\delta\phi_k$ .

upper panel of Fig. 3). Then as the field reaches the minimum, the kinetic energy is damped away through a series of high-frequency oscillations. During this oscillatory regime, which is similar to that of the inflaton in the reheating phase, the scalar field perturbations are unstable and exponentially amplified by the background-field oscillations provided that their frequency increases as their amplitude diminishes [11]. This is indeed the case as shown in Fig. 3, where we plot the evolution of  $\phi$  (right upper panel),  $\delta\phi_k$  and  $\delta_{DM}$  for three different wave-numbers,  $k = 10^{-3}, 10^{-2}$  and  $10^{-1}$ . We can see there that an instability occurs roughly at the same time of the first minimum-crossing oscillation, then followed by a second stage of exponential growth at the beginning of the second oscillation. Such unstable modes are similar to those found in [8], in fact by averaging over periods of time larger than the characteristic time of the oscillations, the scalar field behaves effectively as a dark energy fluid with a constant equation of state  $w$ , as the case considered in [8].

## CONSTRAINTS FROM SN IA, CMB AND LSS: A PRELIMINARY ANALYSIS

Coupled models have been tested against cosmological observations in various works [12, 13, 14]. The main conclusion of these analysis is that current measurements of the CMB anisotropy power spectra and the matter power spectrum from galaxy surveys constrain the coupling constant to be  $\beta < 0.01 - 0.1$  depending on the specific model realization. However none of these works have considered the case in which the scalar field evolves in the adiabatic regime. To this purpose we used the lastest SN Ia-UNION dataset compilation [15], WMAP-5 years data [16] and matter power spectrum measurements from SDSS-5 data release [17] to test the viability of a non-minimally coupled scalar field model with power law potential in the adiabatic regime. For this purpose we have specifically set the field evolution to satisfy Eq. (10), implemented the perturbation equations in a properly modified version of the CMBFAST code [18] and run a Markov Chain Monte Carlo likelihood evaluation. For simplicity we assume a flat universe and fix  $\beta = 1$  and  $\alpha = 0.2$ , while let all other parameters to vary. Here we simply aimed to test whether an adiabatic solution can provide compatible fit to the data, and we leave to a future study a more detailed analysis of the full model parameter space, including  $\alpha$  and  $\beta$ . The marginalized 1D likelihood are shown in Fig. 4 and the best fit value and  $1\sigma$  errors are:  $\Omega_{DM} = 0.222 \pm 0.025$ ,  $\Omega_b h^2 = 0.02226 \pm 0.00068$ ,  $h = 0.75 \pm 0.03$ ,  $\tau = 0.072 \pm 0.017$ ,  $n_s = 0.89 \pm 0.02$ ,



**FIGURE 4.** Marginalized 1D likelihoods.

$A_s = 0.75 \pm 0.03$  and  $b = 1.44 \pm 0.49$ . These constraints are consistent with those derived for  $\Lambda$ CDM cosmologies, the total  $\chi^2$  is close to that of the vanilla  $\Lambda$ CDM model such the two scenarios are statistically indistinguishable. As shown in [10] differences due to the scalar interaction may arise on the small scale clustering of dark matter. Overall this preliminary analysis suggests that adiabatic coupled models with natural coupling are consistent with current cosmological observations.

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